Preferential Semantics As the Basis for Defeasible Reasoning in Ontologies

Tommie Meyer

Department of Computer Science
University of Cape Town
and
Centre for Artificial Intelligence Research (CAIR)
South Africa

tmeyer@cs.uct.ac.za
http://cair.meraka.org.za/~tmeyer

DARe 2015

Outline

1. Defeasible reasoning (informally)
2. Description logics (DLs)
3. The KLM approach to defeasible reasoning
4. Extended to other forms of defeasible reasoning
5. Entailment for defeasible reasoning

1. Defeasible Reasoning (Informally)

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

Biomedical exampes

Situs Inversus [Rector 2004]

Human hearts are usually located on the left-hand side of the body. In humans with situs inversus the heart is located on the right-hand side.

Eukaryotic cells [Stevens et al. 2007]

Eukaryotic cells are by definition those with a proper nucleus. However, some cells that lack a proper nucleus, e.g. mammalian red blood cells, are considered eukaryotic, too.

Policy formulation and legal knowledge

Policy Refinement [Bonatti et al. 2010]

Users should not access confidential files. Staff members should access confidential files. Blacklisted staff cannot access confidential files

Overriding legal knowledge [Bonatti et al. 2010]

Article 2 of DM.270/09 is modified as follows: "..."

Defeasibility vs nonmonotonicity

[Nonmonotonicity - Britz et al. 2013]

Private lawyers normally have only paying clients

Normal private lawyers have only clients who pay, but we allow for exceptions

Defeasibility [Britz et al. 2013]

Private lawyers (always) have clients that normally pay

The normal clients of all private lawyers pay, but some clients of every private lawyer do not (pro bono work)

2. Description Logics

Description logics

- A family of knowledge representation languages
- More expressive than propositional logic (sometimes)
- Decidable fragments of first-order logic
- Useful fragments of first-order logic
- Frequently used for representing ontologies

The description logic $\mathcal{ALC}$

Concepts

- Concept names: *Male*, *Female*, *Father*, *Mother*, *Parent*, $\bot$
- Roles names: *parentOf*
- Boolean concept constructors: and ($\sqcap$), or ($\sqcup$), not ($\neg$)
- Value restriction ($\forall$), Existential restriction ($\exists$)

Examples

- $\mathit{Mother} \sqcup \mathit{Father}$, $\neg \mathit{Parent}$, $\neg \mathit{Parent} \sqcap \mathit{Male}$
- $\exists \mathit{parentOf}.(\mathit{Male} \sqcup \mathit{Female})$
- $\forall \mathit{parentOf}.\mathit{Male}$

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

The description logic $\mathcal{ALC}$

Formal semantics

- Interpretation $\mathcal{I}$: non-empty domain $\Delta^{\mathcal{I}}$ and a function $\cdot^{\mathcal{I}}$
- $\cdot^{\mathcal{I}}$ maps concept name A to a subset $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, $\bot^{\mathcal{I}} = \emptyset$
- $\cdot^{\mathcal{I}}$ maps role name r to a binary relation $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
- $(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$, $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$, $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
- $(\forall r.C)^{\mathcal{I}} = \{x \mid \ y \in C^{\mathcal{I}}$ for every y s.t. $(x, y) \in r^{\mathcal{I}}\}$
- $(\exists r.C)^{\mathcal{I}} = \{x \mid \ y \in C^{\mathcal{I}}$ for some y s.t. $(x, y) \in r^{\mathcal{I}}\}$

The description logic $\mathcal{ALC}$

Terminological information (TBox)

- $C \sqsubseteq D$ and $C \equiv D$
- $C \sqsubseteq D$ is true in $\mathcal{I}$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
- $C \equiv D$ is true in $\mathcal{I}$ iff $C^{\mathcal{I}} = D^{\mathcal{I}}$

Examples

- $\mathit{Person} \equiv \mathit{Male} \sqcup \mathit{Female}$, $\mathit{Male} \sqsubseteq \neg \mathit{Female}$
- $\mathit{Mother} \equiv \mathit{Parent} \sqcap \mathit{Female}$
- $\mathit{Parent} \equiv \exists \mathit{parentOf}.(\mathit{Male} \sqcup \mathit{Female})$
- $\mathit{MotherWithOnlySons} \equiv \mathit{Mother} \sqcap \forall \mathit{parentOf}.\mathit{Male}$

The description logic $\mathcal{ALC}$

Reasoning in DLs

For a TBox $\mathcal{T}$ and a Tbox statement $C \sqsubseteq D$, $\mathcal{T} \models C \sqsubseteq D$?

Example

$\mathcal{T} = \{Person \equiv Male \sqcup Female, Male \sqsubseteq \neg Female,$
$Mother \equiv Parent \sqcap Female,$
$Parent \equiv \exists parentOf.(Male \sqcup Female),$
$MotherWithOnlySons \equiv Mother \sqcap \forall parentOf.Male\}$

$\mathcal{T} \models Mother \sqsubseteq \neg Male$, $\mathcal{T} \not\models Person \sqsubseteq Male$, $\mathcal{T} \not\models Person \sqsubseteq \neg Male$

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

3. The KLM approach to defeasible reasoning

Defeasible subsumption

Example

$$\mathcal{T} = \{ViralMeningitis \sqsubseteq Meningitis,$$
$$BacterialMeningitis \sqsubseteq Meningitis, Meningitis \sqsubseteq \neg Fatal,$$
$$BacterialMeningitis \sqsubseteq Fatal,$$

$$\mathcal{T} \models BacterialMeningitis \sqsubseteq Fatal \sqcap \neg Fatal$$

Problem

- Classical subsumption is monotonic

Defeasible subsumption: $\mathrel{\sqsubset\!\!\!\sim}$

Not expressive enough

Extension with a defeasible version of subsumption: $\mathrel{\sqsubset\!\!\!\sim}$

Example

$$\mathcal{T} = \{\mathit{ViralMeningitis} \sqsubseteq \mathit{Meningitis},$$
$$\mathit{BacterialMeningitis} \sqsubseteq \mathit{Meningitis}, \mathit{Meningitis} \mathrel{\sqsubset\!\!\!\sim} \neg\mathit{Fatal},$$
$$\mathit{BacterialMeningitis} \mathrel{\sqsubset\!\!\!\sim} \mathit{Fatal}\}$$

$$\mathcal{T} \models \mathit{BacterialMeningitis} \mathrel{\sqsubset\!\!\!\sim} \mathit{Fatal}$$

$$\mathcal{T} \not\models \mathit{BacterialMeningitis} \mathrel{\sqsubset\!\!\!\sim} \neg\mathit{Fatal}$$

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

Rational consequence

Given a TBox $\mathcal{T}$, the set of entailments of $\mathcal{T}$ should contain $\mathcal{T}$ and satisfy the following properties:

(Ref) $C \precsim C$

(LLE) $$\frac{C \equiv D, \quad C \precsim E}{D \precsim E}$$

(And) $$\frac{C \precsim D, \quad C \precsim E}{C \precsim D \sqcap E}$$

(RW) $$\frac{C \precsim D, \quad D \models E}{C \precsim E}$$

(Or) $$\frac{C \precsim E, \quad D \precsim E}{C \sqcup D \precsim E}$$

(CM) $$\frac{C \precsim D, \quad C \precsim E}{C \sqcap E \precsim D}$$

(RM) $$\frac{C \precsim D, \quad C \not\precsim \neg E}{C \sqcap E \precsim D}$$

(Cons) $\top \not\precsim \bot$

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

A preferential semantics

Ranked interpretations

- Extend every first-order interpretation $\mathcal{I}$ with a well-founded total preorder $\preceq$ on the domain of $\mathcal{I}$
- $C \sqsubseteq\!\!\!\sim D$ is satisfied in $(\mathcal{I}, \preceq)$ if and only if the $\preceq$-minimal C-objects are also D-objects
- Ranked interpretations satisfy the properties on the previous slide
- Entailment is then defined in terms of these ranked interpretations $(\mathcal{I}, \preceq)$

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

4. Extensions to other forms of defeasible reasoning

Other forms of defeasibility

Defeasible Equivalence

- C and D are defeasibly equivalent in a ranked interpretation $\mathcal{R}$ if and only if the $\preceq$-minimal C objects and the $\preceq$-minimal D objects coincide:
  - if and only if $C \precsim D$ in $\mathcal{R}$ and $D \precsim C$ in $\mathcal{R}$

Defeasible Disjointness

- C and D are defeasibly disjoint in a ranked interpretation $\mathcal{R}$ if and only if the intersection of the $\preceq$-minimal C objects and the $\preceq$-minimal D objects are empty:
  - if and only if $C \sqcup D \precsim \neg(C \sqcap D)$ in $\mathcal{R}$

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

Defeasible quantifiers for DLs

Just as $\sqsubset\!\!\!\sim$ is a weakening of $\sqsubseteq$ we consider a weakening $\forall\!\!\!\!\sim$ of $\forall$

A private lawyer

A lawyer with only paying clients: $Lawyer \sqcap \forall hasClient.PayingClient$

A private lawyer who sometimes does pro bono work

A lawyer whose normal clients are paying clients:
$Lawyer \sqcap \forall\!\!\!\!\sim hasClient.PayingClient$

Defeasible quantifiers for DLs

The semantics of $\forall$ w.r.t. a ranked interpretation $\mathcal{R}$

- $(\forall r.C)^{\mathcal{R}} = \{x \in \Delta^{\mathcal{R}} \mid \min_{\preceq_{\mathcal{R}}} r^{\mathcal{R}}(x) \subseteq C^{\mathcal{R}}\}$
- Most normal objects reachable via r must all be elements of C

A strengthening ($\exists\!\!\!\sim$) of $\exists$ w.r.t. a ranked interpretation $\mathcal{R}$

- $(\exists\!\!\!\sim r.C)^{\mathcal{R}} = \{x \in \Delta^{\mathcal{R}} \mid \min_{\preceq_{\mathcal{R}}} r^{\mathcal{R}}(x) \cap C^{\mathcal{R}} \neq \emptyset\}$
- At least one of the most normal objects reachable via r must be an element of C

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

5. Entailment for defeasible reasoning

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

A preferential semantics for entailment

First attempt: Ranked entailment

$\mathcal{T} \models C \precsim D$ iff $C \precsim D$ is satisfied in every ranked model of $\mathcal{T}$

Not sufficient: does not satisfy (RM)

No inheritance of properties:

$$\mathcal{T} = \{\mathit{ViralMeningitis} \sqsubseteq \mathit{Meningitis}, \\ \mathit{BacterialMeningitis} \sqsubseteq \mathit{Meningitis}, \mathit{Meningitis} \precsim \neg\mathit{Fatal}, \\ \mathit{BacterialMeningitis} \precsim \mathit{Fatal}, \mathit{Meningitis} \precsim \mathit{Treatable}\}$$

$$\mathcal{T} \not\models \mathit{ViralMeningitis} \precsim \mathit{Treatable}$$

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

A preferential semantics for entailment

$\mathcal{R} = (\mathcal{I}, \preceq)$

Height $h_{\mathcal{R}}(x)$ of $x \in \Delta^{\mathcal{R}}$ in $\mathcal{R}$ is the length of the (finite) chain $x_0 \prec, \ldots, x$ from x_0 to x, where x_0 is $\preceq$-minimal

Two ranked models $\mathcal{R}$ and $\mathcal{S}$

$\mathcal{R} \ll \mathcal{S}$ if and only if (i) $\mathcal{R}$ and $\mathcal{S}$ have the same domain Δ and (ii) for every $x \in \Delta$, $h_{\mathcal{R}}(x) \leq h_{\mathcal{S}}(x)$

Minimal ranked entailment

$C \sqsubseteq\!\!\!\sim D$ is in the minimal ranked entailment of $\mathcal{T}$ if and only if it is true in all $\ll$-minimal ranked models of $\mathcal{T}$

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

Minimal ranked entailment

- Satisfies all properties of rational consequence
- Corresponds to what KLM refers to as rational closure
- Can be reduced to classical entailment

Supports inheritance of properties

$$\mathcal{T} = \{\mathit{ViralMeningitis} \sqsubseteq \mathit{Meningitis}, \mathit{BacterialMeningitis} \sqsubseteq \mathit{Meningitis}, \mathit{Meningitis} \precsim \neg \mathit{Fatal}, \mathit{BacterialMeningitis} \precsim \mathit{Fatal}, \mathit{Meningitis} \precsim \mathit{Treatable}\}$$

$$\mathcal{T} \models \mathit{ViralMeningitis} \precsim \mathit{Treatable}$$

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

Computing minimal ranked entailment

Compute an exceptionality ranking of elements of $\mathcal{T}$ (simplification)

$\overline{\mathcal{E}_i}$ is the "classical" version of $\mathcal{E}_i$; $\mathcal{E}_0 = \mathcal{T}$

$\mathcal{E}_{i+1} = \{C \sqsubseteq D \in \mathcal{E}_i\} \cup \{C \precsim D \in \mathcal{E}_i \mid \overline{\mathcal{E}_i} \models C \sqsubseteq \bot\}$ for $i \geq 0$

There is a smallest k such that $\mathcal{E}_k = \mathcal{E}_{k+1}$

$\mathcal{T}_0 = \mathcal{E}_k$ and $\mathcal{T}_i = \mathcal{E}_{k-i} \setminus \mathcal{E}_{k-i+1}$ for $1 \leq i \leq k$

Intuition

- $\mathcal{E}_{i+1}$ contains "exceptional" axioms in $\mathcal{E}_i$ relative to $\mathcal{E}_i$
- "Exceptionality" corresponds roughly to specificity
- Produces a ranking of axioms corresponding to importance
- More specific axioms are more important
- Classical axioms are the most important

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

Computing minimal ranked entailment

Exceptionality ranking

$\mathcal{T}_2 = \{Meningitis \mathrel{\sqsubset\!\!\!\sim} \neg Fatal, Meningitis \mathrel{\sqsubset\!\!\!\sim} Treatable\}$
$\mathcal{T}_1 = \{BacterialMeningitis \mathrel{\sqsubset\!\!\!\sim} Fatal\}$
$\mathcal{T}_0 = \{BacterialMeningitis \sqsubseteq Meningitis,$
$ViralMeningitis \sqsubseteq Meningitis\}$

Make "classical"

$\overline{\mathcal{T}_2} = \{Meningitis \sqsubseteq \neg Fatal, Meningitis \sqsubseteq Treatable\}$
$\overline{\mathcal{T}_1} = \{BacterialMeningitis \sqsubseteq Fatal\}$
$\mathcal{T}_0 = \{BacterialMeningitis \sqsubseteq Meningitis,$
$ViralMeningitis \sqsubseteq Meningitis\}$

Computing minimal ranked entailment:

$\mathcal{T} \models \mathit{ViralMeningitis} \sqsubsetsim \mathit{Treatable}$

Find the largest i such that $\mathcal{T}_0 \cup \ldots \overline{\mathcal{T}_i} \not\models \mathit{ViralMeningitis} \sqsubseteq \bot$

$\mathcal{T}_0 \cup \overline{\mathcal{T}_1} \cup \overline{\mathcal{T}_2}$

$\overline{\mathcal{T}_2} = \{\mathit{Meningitis} \sqsubseteq \neg \mathit{Fatal}, \mathit{Meningitis} \sqsubseteq \mathit{Treatable}\}$
$\overline{\mathcal{T}_1} = \{\mathit{BacterialMeningitis} \sqsubseteq \mathit{Fatal}\}$
$\mathcal{T}_0 = \{\mathit{BacterialMeningitis} \sqsubseteq \mathit{Meningitis}, \mathit{ViralMeningitis} \sqsubseteq \mathit{Meningitis}\}$

Check if $\mathcal{T}_0 \cup \overline{\mathcal{T}_1} \cup \overline{\mathcal{T}_2} \models \mathit{ViralMeningitis} \sqsubseteq \mathit{Treatable}$ ✓

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

Computing minimal ranked entailment:

$\mathcal{T} \models BacterialMeningitis \precsim Treatable$

Find the largest i **such that** $\mathcal{T}_0 \cup \ldots \overline{\mathcal{T}_i} \not\models BacterialMeningitis \sqsubseteq \bot$

$\mathcal{T}_0 \cup \overline{\mathcal{T}_1}$

~~$\overline{\mathcal{T}_2} = \{Meningitis \sqsubseteq \neg Fatal, Meningitis \sqsubseteq Treatable\}$~~

$\overline{\mathcal{T}_1} = \{BacterialMeningitis \sqsubseteq Fatal\}$

$\mathcal{T}_0 = \{BacterialMeningitis \sqsubseteq Meningitis, ViralMeningitis \sqsubseteq Meningitis\}$

Check if $\mathcal{T}_0 \cup \overline{\mathcal{T}_1} \models BacterialMeningitis \sqsubseteq Treatable$ ✗

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

Different forms of entailment

Minimal ranked entailment: or prototypical reasoning

- The “most conservative” form of rational consequence
- Syntax independent
- Defeasible subsumption applies to typical situations—in all other cases “all bets are off”

$$\mathcal{T} = \{\mathit{ViralMeningitis} \sqsubseteq \mathit{Meningitis}, \mathit{BacterialMeningitis} \sqsubseteq \mathit{Meningitis}, \mathit{Meningitis} \mathrel{\underset{\sim}{\sqsubset}} \neg\mathit{Fatal}, \mathit{BacterialMeningitis} \mathrel{\underset{\sim}{\sqsubset}} \mathit{Fatal}, \mathit{Meningitis} \mathrel{\underset{\sim}{\sqsubset}} \mathit{Treatable}\}$$

$$\mathcal{T} \models \mathit{ViralMeningitis} \mathrel{\underset{\sim}{\sqsubset}} \mathit{Treatable}$$

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

Different forms of entailment

Lexicographic closure: or presumptive reasoning

- More "adventurous" than prototypical reasoning
- Syntax dependent
- Defeasible subsumption means "in the absence of contradictory information, the subsumption holds"

$$\mathcal{T} = \{ \mathit{ViralMeningitis} \sqsubseteq \mathit{Meningitis}, \\ \mathit{BacterialMeningitis} \sqsubseteq \mathit{Meningitis}, \mathit{Meningitis} \mathrel{\underset{\sim}{\sqsubset}} \neg \mathit{Fatal}, \\ \mathit{BacterialMeningitis} \mathrel{\underset{\sim}{\sqsubset}} \mathit{Fatal}, \mathit{Meningitis} \mathrel{\underset{\sim}{\sqsubset}} \mathit{Treatable} \}$$

$$\mathcal{T} \models \mathit{BacterialMeningitis} \mathrel{\underset{\sim}{\sqsubset}} \mathit{Treatable}$$

CENTRE FOR ARTIFICIAL INTELLIGENCE RESEARCH

Computing presumptive reasoning

Refined ranking of sentences by exceptionality

$\mathcal{T}_3 = \{Meningitis \mathrel{\sqsubset\!\!\!\sim} \neg Fatal, Meningitis \mathrel{\sqsubset\!\!\!\sim} Treatable\}$
$\mathcal{T}_2 = \{Meningitis \mathrel{\sqsubset\!\!\!\sim} \neg Fatal \sqcup Treatable\}$
$\mathcal{T}_1 = \{BacterialMeningitis \mathrel{\sqsubset\!\!\!\sim} Fatal\}$
$\mathcal{T}_0 = \{BacterialMeningitis \sqsubseteq Meningitis,$
$ViralMeningitis \sqsubseteq Meningitis\}$

Make “classical”

$\overline{\mathcal{T}_3} = \{Meningitis \sqsubseteq \neg Fatal, Meningitis \sqsubseteq Treatable\}$
$\overline{\mathcal{T}_2} = \{Meningitis \sqsubseteq \neg Fatal \sqcup Treatable\}$
$\overline{\mathcal{T}_1} = \{BacterialMeningitis \sqsubseteq Fatal\}$
$\mathcal{T}_0 = \{BacterialMeningitis \sqsubseteq Meningitis, ViralMeningitis \sqsubseteq Meningitis\}$

Computing presumptive reasoning:

$\mathcal{T} \models BacterialMeningitis \precsim Treatable$

Find the largest i such that $\mathcal{T}_0 \cup \ldots \overline{\mathcal{T}_i} \not\models BacterialMeningitis \sqsubseteq \bot$

$\mathcal{T}_0 \cup \overline{\mathcal{T}_1} \cup \overline{\mathcal{T}_2}$

~~$\overline{\mathcal{T}_3} = \{Meningitis \sqsubseteq \neg Fatal, Meningitis \sqsubseteq Treatable\}$~~

$\overline{\mathcal{T}_2} = \{Meningitis \sqsubseteq \neg Fatal \sqcup Treatable\}$

$\overline{\mathcal{T}_1} = \{BacterialMeningitis \sqsubseteq Fatal\}$

$\mathcal{T}_0 = \{BacterialMeningitis \sqsubseteq Meningitis, ViralMeningitis \sqsubseteq Meningitis\}$

Check if $\mathcal{T}_0 \cup \overline{\mathcal{T}_1} \cup \overline{\mathcal{T}_2} \models BacterialMeningitis \sqsubseteq Treatable$ ✓

Reduction to classical entailment checking

Pre-processing (ranking)

$O(|\mathcal{T}|^3)$ classical entailment checks

Prototypical reasoning

$O(|\mathcal{T}|)$ classical entailment checks

Presumptive reasoning

$O(2^{|\mathcal{T}|})$ classical entailment checks

Summary

- Defeasible subsumption based on a preferential semantics
- Reduction to classical entailment
- Preferential semantics generates axiom preferences
- Other forms of defeasibility (equivalence, disjointness)
- Reduction to defeasible subsumption
- Defeasible quantifiers - no known reduction yet
- Experimental evaluation of prototypical reasoning

Future work

- Optimised implementation
- Evaluation on large ontologies
- Reduction of defeasible quantification to classical case
- Beyond the Tbox to evaluate queries
- Other forms of “rational” entailment